A revision of the theory of modulation and a renewed Riemannian analytical methodology for extended tonality [1]

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Abstract: This paper presents a proposal for a revision of the theory of modulation in two parallel systems of classification, according to the criteria of modulatory impact and modulatory facture. This proposal is presented together with a series of theoretical definitions and explanations that serve as grounds for the proposal of a renewed Riemannian methodology for the analysis of nineteenth-century extended-tonality works. The paper tries to demonstrate the historical grounds of the analytical propositions made, and specially highlights their ability to graph the harmonic language of nineteenth-century extended tonality by means of a few analytical examples taken from repertoire.

Keywords: Music Analysis; Theory of Modulation; Music Structure; Tonal Harmony.

1. Introduction

The present paper intends to anticipate to the general academic public the grounds for a proposal of an analytical methodology for Tonal Music, specially for its chromatic type from the second half of the nineteenth century. Included in these grounds are a series of propositions of taxonomies, nomenclatures and analytical symbologies that are currently being developed as part of a larger research project of formulation of a structural model for nineteenth-century tonality that aims at finding, explaining and graphing the constructive coherence of the repertoire of the second half of the nineteenth century in a clear, synthetic and pedagogical way, grounded on a methodology based on the critical revision of historical theoretical bibliography of that time. In the course of this research, the need was felt for an expressive return to the original nineteenth-century sources, dissecting musical texts from the repertoire of several time periods, re-assembling and re-working the concepts of the nature of chords, the nature of

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counterpoint, the functionality of Harmony and the logic of the harmonic progressions. Since the main interest of this research concerns the late Romantic repertoire, the most important sources have been, either directly or mediated by contemporary musicologists, texts from the nineteenth century and beginning of the twentieth century by authors such as Gottfried Weber (1779-1839), Anton Reicha (1770-1836), Moritz Hauptmann (1792-1868), François-Joseph Fétis (1784-1871), Arthur von Oettingen (1836-1920), Carl Friedrich Weitzmann (1808-1880), Arnold Schoenberg (1874-1951), Heinrich Schenker (1868-1935), and specially Hugo Riemann (1849-1919). Constructed from this critical revision of the nineteenth-century theoretical imagination, this research intends to propose an alternative way to explain the compositional procedures of Extended Tonality, formalized in the form of a Structural Model which will be presented to the most possible extent as a logical deductive system demonstrated "in geometrical order", as in Spinoza's Ethics, in the most objective and pedagogical way possible. With this research, the goal is not to solely create a pedagogical and theoretical body of texts for the teaching of Musical Structuring, but also to recuperate for the contemporary scholars part of the musical theoretical imagination of the nineteenth century, a body of work which is reasonably unknown to the contemporary traditional methodologies of musical instruction, despite its utmost importance for understanding nineteenth-century repertoire.

A major part of this research involves the finding of recurring paradigmatic traits in the historical repertoire and the research and (re)formulation of models which are able to explain, classify, and interrelate such practices, their origins, properties, developments, and transmutations. The typologies created by these proposed models generate a system of symbols and terminologies that are used as semantic points of access to the typological content. This symbology and terminology become the major and basic tools for the process of formalization and summarization of the comprehension and analytical mapping of Music, becoming the very vehicle of communication and pedagogical transmission of the concepts studied. It is relevant to remark that these typologies of models of musical structural elements, identified by their terminologies and symbologies, living and surviving through the times in several lineages of theoretical works, that form after all the very musical world of imagination of musicians.

A warning is due here that there are some serious pitfalls in this process of theoretical recreation. A first problem is that the concepts, theories, and basic terminologies of Music Theory are not unified and are very often presented differently and sometimes conflictingly by different theorists. For example, I suggest the reading of the interesting comparison made by Zamacois (in ZAMACOIS, 1984, p. 235) of ideas regarding the kinship of tonalities formulated by Riemann, Nikolai Rimsky-Korsakov (1844-1908), and Vincent d'Indy (1851-1931). More specifically, the matter of terminologies is a big problem, for a specific term may have been not only defined differently by different authors, but it may have been used to signify things that are absolutely unrelated. The solution for this problem asks either the redefinition of certain terms that because of their widespread usage cannot be abolished – thus forcing the theorist to pick a specific side, so to speak, in relation to his choices –, or even the creation of absolutely new terminologies, with the hope that a different jargon will not incur in confusions and misunderstandings.

A second problem comes from the pertinent realization that for the effectiveness of a music-theoretical work there is the need for a certain temporal distance from the observer (the theorist) and the observed (the composers of a certain time period and style and their repertoire). It is only with this distancing that it is possible to separate that which is simply an isolated mannerism of an author from that which is a common practice of a community of authors. The problem is that, with this distance, what is gained in vision is lost in proximity with the living musical common practice of that time. This happens because that practice is not entirely preserved in the historical theoretical texts: these usually tended to formalize only the most complicated aspects of the musical language of the period, while matters of more common comprehension were omitted for the lack of need for the expression of things which were more commonly and generally understood (see the commentary on the theoretical work of Tinctoris in JEPPESEN, 1992, p. 9-10, for example). My opinion is that perhaps the best attitude for the resolution of these pitfalls is to accept that maybe we will never be able to find all the missing pieces of this puzzle, and focus our attention to the recurring aspects found in the very musical texts, explaining them in a way that is relevant to the needs of our own times, but without inventing theoretical novelties that are distant from the past world of imagination of that time period. This can be accomplished through the critical study of that which has been partially preserved in the historical treatises, always having in mind that those texts cannot be taken entirely without critic simply because they lack a certain temporal distance from their object of study.

While the definitive formalization of this research is being prepared in the form of a treatise, some of its most important aspects are being presented to the academic community separately in the form of individual papers. This present paper reworks a previous text of mine (BITTENCOURT, 2012) in a more precise, correct and considerably expanded and revised way, and it presents an important fragment of my work concerning a pedagogical (re)organization of the Theory of Modulation – a "reboot", so to speak, of this most important theme of Music Theory –, with recastings of theoretical definitions and a taxonomy prepared according to the concerns and orientations previously described.

This work is strongly based in a former proposal for a nomenclature and analytical symbology (see BITTENCOURT, 2013), which was developed for this research as a basic tool for structural analysis. This taxonomy of modulation, together with the aforementioned symbological reform, forms the main basis for the analytical methodology proposal which makes possible this bigger research of a structural model for nineteenth-century tonality.

2. Some (re)definitions and basic concepts

2.1. Harmonic Structures

A Harmonic Structure, from a generalized viewpoint, is defined in this theoretical proposition as a specific collection of different notes to which we attribute a musical meaning that does not change after suffering operations of rotation, octave displacement or shuffling of its constituent notes, and that serves as a contextualizing element for everything that happens harmonically and melodically during the specific slice of time in which it lives. In the context of repertoire from before the twentieth century, the ideal harmonic structure is a collection of different notes that can best produce a high Tonicity (the measure of the probability of the existence of a common fundamental for the collective frequency spectrum of the sounding notes – see BITTENCOURT, 2011 p. 52-53, OETTINGEN, 1866, p. 27-35, and the notion of "tonalness" in PARNCUTT, 1989, p. 25-26 and 58-59), and a low Roughness (sensory dissonance caused by the beatings resulting from the interaction of the partial frequencies of the collective spectrum of the sounding notes – see PARNCUTT, 1989, p. 25 and 58-59, and HELMHOLTZ, 1875, p. 278). These ideal conditions happen when we divide the diapente interval (3:2) at the point of a sesquiquarta interval (5:4), in other words, this occurs with the major triad and its inversion, the minor triad (see RIEMANN, 1903, pg. 6, and BITTENCOURT, 2011, p. 52-53). As models of perfection and consonance (a combination of high tonicity and low roughness), the perfect major and minor triads have for centuries populated the world of imagination of composers and serve as main contextualizing element for the simultaneity of pitches (see RIEMANN, 1992 p. 87-88). It is worth mentioning here that it is possible to add other notes to the triadic harmonic structures of the dominant and subdominant that, by adding tension to its basic triad, boost the functional definition of those structures: the sevenths and ninths in the dominant structure, and the major sixth in the subdominant structure (see RIEMANN, 1903, p. 55-61).

Traditional Scale-Degree Notation	Functional Notation		
	Bittencourt	Riemann	Mäler
I	$^+$T	T	T
ii	$^+$Sr	Sp	Sp
II = V/V	DD	DD	DD
iii/I	$^+$Ta=Dr	T̸ =Dp	Tg =Dp
III/I	$^+$Ta+	T̸^{v}	TG
IV	$^+$S	S	S
V	D=°D=$^+$D	D	D
vi/I	$^+$Tr	Tp	Tp
VI/I	$^+$Tr+	Tp^v	TP
vii^{ø7}	$^+$D̸^{9}	D̸^{9}	D̸$^9_7$
vii	$^+$Da	D̸	Dg
♭VII	D°r = $^+$SS	°Dp=SS	dP=SS

Traditional Scale-Degree Notation	Functional Notation		
	Bittencourt	Riemann	Mäler
i	°T	°T	t
♭II	°Sa	S̸	sG
ii°	°S^6	°S^6	s^6
III/i	°Tr	°Tp	tR
iii/i	°Tr∘	°Tp^v	tr
iv	°S = Ɑ	°S	s
v	D° = Ƨ	°D	d
VI/i	°Ta	T̸	tG
vi/i	°Ta∘	T̸^{v}	tg
vii°^{7}	°D̸^{9}	D̸$^{9>}$	D̸$^{9>}_7$
VII	Da+	D̸^{v}	DG
♭vii	D°r∘ =°SS	°Dp^v=°SS	dp=ss

Figure 1. Comparison table between the analytical symbologies of the traditional Scale-Degree Theory and the Functional Theory, in three different versions (Bittencourt's, Riemann's, and Mäler's).

In this proposal for an analytical methodology, the harmonic structures are represented according to the proposal for a symbological reform of the Riemannian Functional Harmony described in BITTENCOURT, 2013. This symbological reform takes as point of departure the original symbols created by Riemann (see MICKELSEN, 1977, p. 75-79, and RIEMANN, 1903) and its simplifying revision by Hermann Grabner (1886-1969) (see MICKELSEN, 1977, p. 92-94) in its usual translation from German (see KOELLREUTER, 1980, and BRISOLLA, 2006), together with a series of requisites and specific needs defined by the envisioned analytical methodology (see BITTENCOURT 2013, p. 32-33). Figure 1 shows a comparison table between the symbologies of the traditional Scale-Degree Theory and the

Functional Theory, both in the reformed symbology version used in this research and compared to the historical functional symbols by Riemann and Wilhelm Mäler (1902-1976). The references used for this table were: WEBER, 1851, and GAULDIN, 2004 (scale-degree notation); BITTENCOURT, 2013 (Bittencourt's functional notation); RIEMANN, 1903, REHDING, 2003, and MICKELSEN, 1977 (Riemann's functional notation); LA MOTTE, 1998 (Mäler's notation).

2.2. The Tonal Formula as a Master Framework

Great music theorists such as Heinrich Schenker and Hugo Riemann searched after the idea of a Fundamental Structure (a "*Ursatz*", in Schenker's jargon) which would serve as a master framework for every piece of musical fabric. Inspired by the Schenkerian model (the "*Urlinie*" attached to the "*Bassbrechung*" or "*GrundBassbrechung*", linearly forming a I-V-I; see KATZ, 1935, p. 314, and PANKHURST, 2008, p. 54) and by the Riemannian model (his "*große Cadenz*", the I-IV-I in conjunction with I-V-I; see MOONEY, 2000, p. 84), the present proposal for a model of a fundamental structure is based on the ubiquitous I-IV-V-I progression which gives rise to the three basic traditional cadential models: the full cadence, the half cadence, and the deceptive cadence (also known as false or interrupted cadence). The choice behind the usage of the term "formula" instead of "cadence" aims at eliminating the terminal connotation of the term cadence, directing the focus of our attention beyond the instants of the phrase endings, and casting the notion of a constructive model which is to be respected by all harmonic progressions prior to the twentieth century.

Similar to Schenker's opinion, in which tonality is seen as an expansional horizontal transformation of the perfect triad (see KATZ, 1935, p. 313-315), the present formalization of the concept of a Fundamental Structure also represents a trajectory of expansion and contraction of the same initial sound (the tonic), but described in a manner which is not exactly melodically linear as in Schenker's theories. The principle here is to define the idea of a central tonal place (the aforementioned initial sound, the tonic) by means of an initial movement of departure from that place, followed by a refutation of this departure, with a logical returning conduction to the beginning. Traditionally, the idea of tonal center (thesis, affirmation) is associated with the harmonic function of Tonic, the idea of departure of the center (antithesis, conflict) with the function of subdominant, and the idea of refutation of departure and agent of the returning conduction to the center (synthesis) is associated with the function of Dominant (see MOONEY, 2000, p. 84-85, and SCHOENBERG, 1995, p. 311).

The present proposal of definition of the idea of Fundamental Structure, which is clearly of Riemannian inspiration (see REHDING, 2003, p. 47), is therefore functionally represented by the progression T-S-D-T, which brings about exactly this motion of departure from an arbitrary center and consequent return to it. Initially, this model gives rise to the three basic versions of that which is here called a Tonal Formula, one version representing a complete "landing" motion towards the center, another representing a model in which the return to the center is only suggested but not completed, and yet another in which the conduction to the center is surprised by a "landing" in a place different than expected. Thus, the schematic formalization of these basic versions of the tonal formula is as follows:

- The Complete Tonal Formula, which is based on the traditional full cadence:[Tx]-[Sx]-D-T ;
- The Incomplete Tonal Formula, which is based on the traditional half cadence: [Tx]-[Sx]-D ;

- The Deceptive Tonal Formula, which is based on the traditional deceptive cadence: [Tx]-[Sx]-D-"*surprise*", in which the "surprise" can be implemented with any harmonic structure which is different than the main tonic, preferably structures which contain a reasonable amount of resolution tones for the tensions of the dominant.

In these presented schemes, the brackets "[ ]" mean that the respective area of the formula is optional and it can be omitted; the "x" denotes the possibility that the respective area of the formula may be implemented by a substitute structure of the main harmonic structure in question, such as a sixth substitute (Riemann's *parallelklang*, or relative substitute in the translation from German) or a leading-tone substitute (Riemann's *leittonwechselklang*, or anti-relative substitute) (see RIEMANN, 1903, p. 71 and p. 80). Do note that according to these given definitions, the participation of the main tonic triad in a formula is not indispensable.

Nonetheless, the tonal formula is not a simple common cadential progression. It is a basic principle, a master framework, which can be stretched in time by the action of structural prolongations attached to the main harmonic structures of the formula as prefixes or suffixes, and by means of melodic prolongations of the main structural tones of the harmonic structures, namely the ubiquitous foreign or non-structural tones; and thus the framework blooms towards the very foreground surface of the musical fabric. It is like this that Riemann conceived the principle of "Musical Logic" (see REHDING, 2003, p. 47). In a certain way, here are conjoined both the main idea of prolongation, which was of utmost importance to Schenker, and the principle of Harmonic Function.

Eventually, it is possible to admit the use of substitute harmonic structures also for the formula areas of the dominant in the complete and incomplete formulas, with the participation of the Extraordinary Dominants and the Regnante. The Regnante is the inverted dominant, a dualist term by Oettingen (see OETTINGEN, 1866, p. 67), which is represented by the triad of iv, always as a minor chord. The regnante has the same functional properties of the dominant, but with the natural resolutions and voice leading in an inverted way. It is represented in the proposal for an analytical symbology here used by the reverse of the functional symbol for the dominant (see BITTENCOURT, 2013, p. 36). The use of mirrored versions of the letters D e S to denote the Regnante and the Supra-Regnante (the triad of v, always as a minor triad; see OETTINGEN, 1866, p. 67) was in a certain way inspired in the symbology by Sigfrid Karg-Elert (1877-1933) (see MICKELSEN, 1977, p. 90).

In this proposed model, an Extraordinary Dominant is a substitute harmony of the main dominant, which is constructed and effectively connected to the tonic harmony according to procedures based on the voice-leading schemes found in the most common cadences, specially the traditional deceptive cadences and the cadential six-four (see BITTENCOURT, 2013, p. 39-40). Of great importance for the study of the repertoire of the second half of the nineteenth century, the concept of this collection of accessory dominants is inspired by ideas for unorthodox interconnections between chords via basic models of voice leading suggested by Karl Friedrich Weitzmann in his *Harmoniesystem* of 1860 (see RUDD, 1992, p. 65-69). As a matter of fact, such connections also deserved Schenker's attention (see the cases of tonicalization in SCHENKER, 1954, p. 265-276). Figure 4 includes examples of the use of extraordinary dominants and regnants.

In this sense, the tonal formula represents exactly this implementation act of a tonal center, a necessary act of demonstration of a tonal place which models in a backwards way the "habits of the ear" postulated by Gottfried Weber (WEBER, 1851, p. 345). Thinking in this manner, without the performance of a tonal formula there can't be the idea of a tonality. Thus we posit the proposition that every harmonic structure of a musical work from before the twentieth century participates ineluctably of a tonal formula, and it is duly inserted in this formula and contextualized by it.

In this present proposal for an analytical methodology, a tonal formula is represented by marking its extension with a horizontal line. Over this line we will place the functional symbols of the instantiated harmonic structures and under it the symbol of the tonal region defined by the formula (see figure 2), according to the proposal for a symbological reform of the Riemannian Functional Harmony presented in BITTENCOURT, 2013, and also according to a series of additional conventions that will be explained further down this paper.

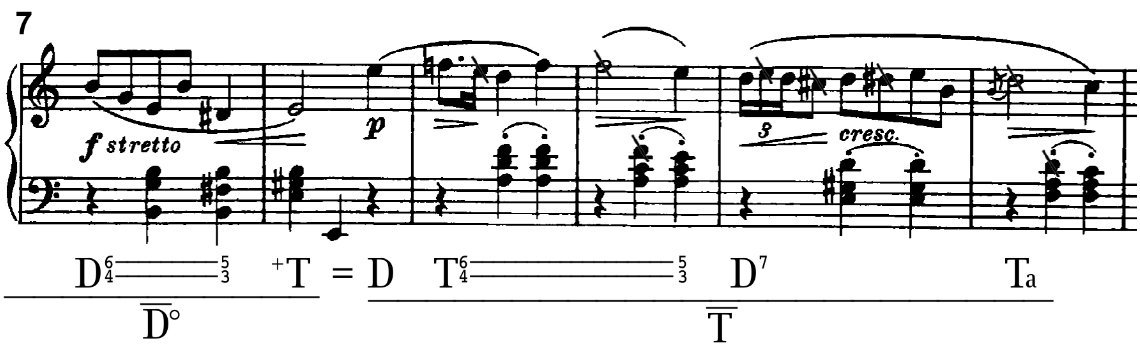

Figure 2. Example of the analytical notation of tonal formulas.
Fréderic Chopin (1810-1849), Mazurka Op. 7 nº 2 in A minor (composed between 1830-31), measures 7 to 12.

2.3. Tonal Region and Harmonic Field

Comprehensively worked out by Schoenberg (see SCHOENBERG, 1969, p. 19-34), the concept of Tonal Region is defined in this proposal for a model of tonality as a diatonic tonal place (major or minor) which is instantiated in the perception of the listener by means of the performance of a tonal formula, in one of its three basic versions (see last item). The concept of region is of the utmost importance to the comprehension of the principle of Monotonality (see SCHOENBERG, 1969, p. 19), according to which a musical work would be structured in one and only one tonality, being this tonality understood not as a simple diatonic place but as a universe of regions hierarchically oriented according to relationships of proximity and separation around one central region which at the end gives name to that tonality (see the diagrams in SCHOENBERG, 1969, p. 20 and 30).

The Harmonic Field of a region is defined in this proposal as the collection of harmonic structures that are available for the implementation of tonal formulas in that region. It includes initially the harmonic structures of tonic (I), subdominant (IV), and dominant (V), with the further addition of their respective sixth substitutes and leading-tone substitutes (see RIEMANN, 1903, p. 71 and p. 80). In the minor mode, the presence of the dominant, which is always a major triad, causes still the need for the addition of the modal parallels of the subdominant (IV instead of iv) and of the dominant (v instead of V) to the harmonic field, also together with their sixth and leading-tone substitutes. This is done to implement the diatonic passages referent to the melodic ascending and descending versions of the minor scale, respectively. The collection of notes which is defined and presented by the main pillar triads of the tonality forms the Diatonic Space (or Scale) of the region in question. It is interesting to remark here that this idea of forming the major and minor scales out of the main triads of I, IV, and V is quite old and it has been mentioned by theorists such as Heinrich Christoph Koch (1749–1816) (see RUDD, 1992, p. 27-29).

In this proposal for an analytical methodology, considering the concept of monotonality, a region is symbolized by the functional symbol that the tonic triad of this region would have in the harmonic field of the central tonal anchor (for the concept of Anchor, see item 3.1.2.), with the further addition of a horizontal dash above its symbol (see BITTENCOURT, 2013, p. 41). This horizontal dash serves to distinguish the symbol used for regions from the ones used for harmonic structures (see figure 2).

3. A taxonomical (re)creation of the Theory of Modulation

Modulation is one of the most important structuring aspects of Western Music, and its importance for the study of Harmony and Music Structure is a notorious fact. In a wide and general sense (and I here stress the presence of the word "general"), this present proposal defines the term "Modulation" as being every procedure that effects the mixture of harmonic structures issued from different harmonic fields. This brutal generalization may initially be seen as a bit shocking, but it nonetheless proves to be extremely useful, for it is from this very generalization that the present taxonomical proposal springs.

3.1. The Modulatory Impact as a criterion for classification

Since every harmonic field serves the purpose of implementing the formation of a specific tonal center, a modulation (in the aforementioned definition) involves a certain perception of displacement of tonal center. The intensity of this tonal center displacement, which we will here call Modulatory Impact, depends on the exact manner in which this modulation is implemented, and the measurement of this impact serves as a most important classifying criterion for modulation, with a significant impact in the analytical comprehension of musical structures. This taxonomical proposal recycles parts of concepts already worked out by several nineteenth-century theorists (such as François-Auguste Gevaert (1828-1908) apud ZAMACOIS 1984, 1988, 1990), but with a redefined, expanded and/or re-delimited terminology.

Considering the diverse ways in which a modulation can impact our perception of tonal center, and also considering the "*Principium Inertiae*" posited by Gottfried Weber (see WEBER, 1851, p. 333-334), there are the following scenarios in which the modulatory procedure, in increasing order of modulatory impact: a) does not effectively defy the authority of the tonal center in effect; b) creates a temporary deviation of tonal focus but in a way that always promotes a periodical return to a basic recurring center, which acquires the status of a Tonal Anchor; c) causes a transport of this tonal anchor to another place (and this is the traditionally most accepted meaning of the term "modulation"); d) destroys the feeling for a tonal anchor by promoting center transfers from one point immediately to another without the mediating recourse of a recurring tonal anchor; and e) obliterates the feeling of a center by means of a progression made out exclusively of harmonic structures that individually form practically alone their own tonal formula, each one pointing towards a different region. Thus, according to its modulatory impact, a modulation will be classified in 5 basic types, namely: the Tonicization, the Intratonal Modulation, the Extratonal Modulation (or Modulation Proper), the Crossing, and the Wander.

3.1.1. The Tonicization

Comprehensively worked out in detail by several theorists (see "tonicalization" in SCHENKER 1906, p. 256-261), the Tonicization is defined in this present model as a modulatory procedure in which a harmonic structure belonging to the harmonic field of the region activated by the tonal formula is prefixally prolonged via another harmonic structure borrowed from the harmonic field whose tonic is the very structure being prolonged. Here the mixture of elements from different harmonic fields is smooth enough so that a real displacement from the tonal center established by the formula does not happen. More commonly, this type of modulation involves the use of applied dominants, traditionally also known as secondary dominants (see BRISOLLA 2006, p. 63), a type of prefixal structural prolongation that introduces a harmonic structure by means of its own dominant, in other words, by means of the dominant from the harmonic field whose tonic is the "tonicized" element (namely, the structure being prefixally prolonged). It is important to stress the fact that the tonicized structure does not lose the harmonic function that it performs inside the tonal formula that activated the original region. It is precisely because of this that we can say the tonicization is a procedure with zero modulatory impact, for it does not defeat the "*Principium Inertiae*" (WEBER, 1851, p. 333-334).

In this present proposal for an analytical methodology, the tonicization is indicated using the symbol for the region of the borrowed harmonic field, placed in parentheses underneath the functional analytical notation of the tonicizing harmonic structure, according to the symbology proposal described in BITTENCOURT 2013, p. 41-42 (see figure 3). Since every symbol in parentheses always refers to a region, it is deemed unnecessary to use the horizontal dash above that symbol.

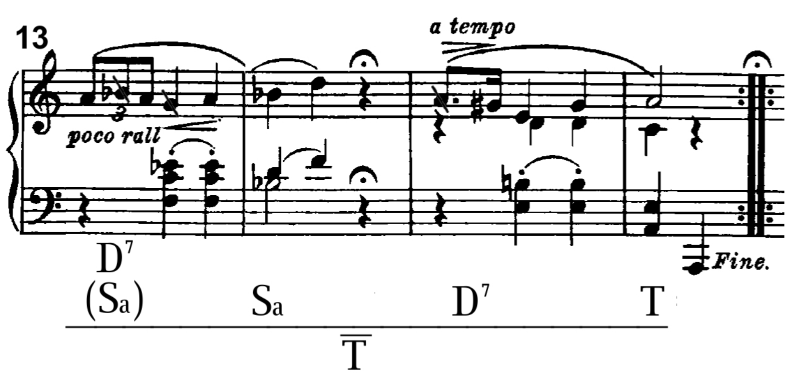

Figure 3. Example of the analytical notation of tonicizations.
Fréderic Chopin, Mazurka Op. 7 nº 2 in A minor, measures 13 to 16.

3.1.2. The Intratonal Modulation

The Intratonal Modulation is defined in this present model as a modulatory procedure in which tonal formulas in different neighboring regions alternate in a manner to create the impression of the existence of a central, basic and recurring region, the Tonal Anchor, which imposes itself as the effective center.

Here, the modulation has enough impact to dislodge the tonal center to a neighboring region, but only in a temporary fashion, from where it soon returns to the anchor region. This is the most characteristic behavior of a traditional tonal musical fabric, becoming the basic model for tonal stability. It is as if the other neighboring tonalities were to serve as "advanced outposts" of the central tonality, stretching its domains and helping to delimit its frontiers via the proposition of diatonic contrasts. It is at this point that the notion of a Greater-Tonality is constructed (a "Tonality with capital T"), a universe of tonalities in which these are hierarchically organized around a single main tonality according to relationships of proximity and separation. Indeed, this is the same spirit behind the principle of Monotonality described by Schoenberg (SCHOENBERG 1969, p. 19-20).

This term "intratonal modulation" has already been used by Gevaert (see ZAMACOIS 1988, p. 133-134), although in a different sense (given the musical examples offered by Zamacois), and more in the context of that which in this present model is called "tonicization". The idea of "intratonal" here explained keeps Gevaert's idea of an action that results in a brief and temporary transference of tonal center, but now considering the contextualizing element of Schoenberg's "monotonality". In other words, this transference of center is brief because there exists a clear contextual intention of keeping a tonal anchor, and it is precisely because of this that "every digression from the tonic is considered to be still within the tonality, whether directly or indirectly, closely or remotely related" (SCHOENBERG 1969, p. 19).

In the present proposal for an analytical methodology, the intratonal modulation is indicated by the very region symbol which is included underneath the horizontal line that delimits the tonal formula. The functional symbol used must specifically reveal the kin relationship that the region of the analyzed formula maintains with the tonal anchor of the music section under consideration (see figure 2). If the tonal anchor of the music section under consideration is different from the main tonality of the work, one should further add, inside parentheses and below the basic symbol for the region, another notation containing the functional symbol that specifically reveals the kin relationship maintained between the tonal anchor of the music fragment in question and the main tonality of the work. In figure 4, for example, which shows the beginning of the second theme from the first movement of the Ninth Symphony in D minor by Anton Bruckner (1824-1896), the regions of the formulas in A major, E major, A minor, G major, and F sharp major are identified in relation to A major, which is the tonal anchor of the passage in question. The notations inside parentheses underneath the notation for the region reveal the relationship that the anchor (A major) maintains with the main key of the movement (D minor).

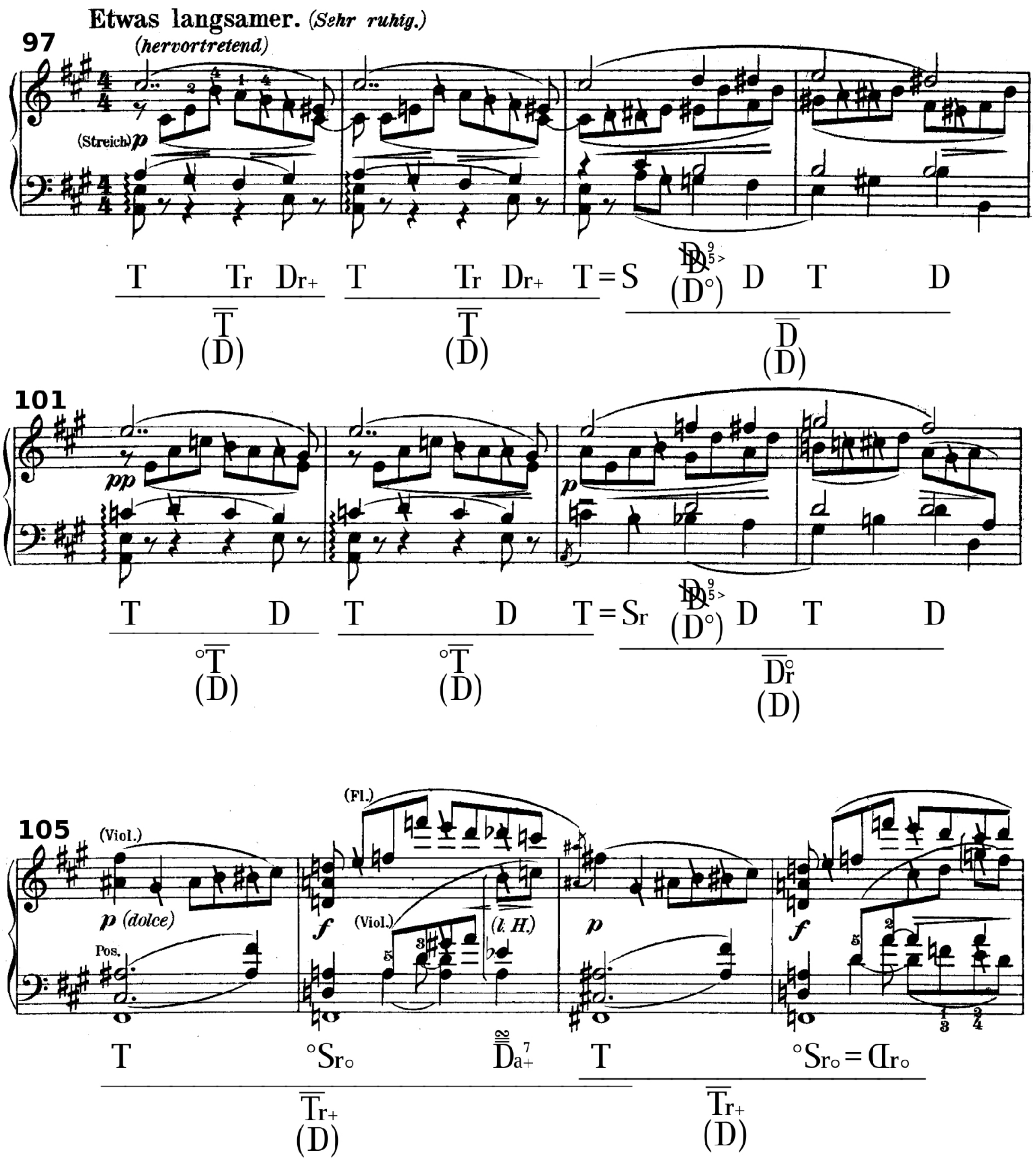

Figure 4. Example of analytical notations for intratonal and extratonal modulations.
Anton Bruckner (1824-1896), Ninth Symphony, in D minor (1887-96), first movement, measures 97-108.

3.1.3. The Extratonal Modulation.

The Extratonal Modulation, in this model, refers to the procedure of displacement of the central axis of intratonal modulation (the tonal anchor) to another place. It would have been possible to call this type "Modulation Proper", considering that this term denotes that which theorists such as Schoenberg commonly conceive as the true meaning of the word "modulation":

> "One should not speak of modulation unless a tonality has been abandoned definitely and for a considerable time, and another tonality has been established harmonically as well as thematically".
> (SCHOENBERG, 1969, p. 19).

Here the modulation impact is capable to dislodge the tonal anchor to another locality, around which new sessions of intratonal modulation develop. This is yet another term that revisits ideas by Gevaert (see ZAMACOIS 1984, p. 194), although with a recast and enlarged meaning.

In the present proposal for an analytical methodology, the extratonal modulation is graphed by that very symbol located inside parentheses underneath the basic notation for the region, according to the explanation contained in the last item. The example from figure 4 indicates that an extratonal modulation has occurred, for the tonal anchor of the theme under discussion is different from the main tonality of the movement (D minor).

This analytical symbology allows for the differentiation of situations in which the same tonality would represent, in different sections of a work, different regional relationships according to the tonal anchor of each context. For example, in a musical fragment anchored in C major, A minor would represent the region of the relative tonic of that anchor, but if the anchor migrates to G major, the same A minor would then represent the region of the relative subdominant of the new main region. I am of the opinion that it is very important for the structural comprehension of Music that these distinctions be clearly illuminated and graphed by the analytical symbology.

3.1.4. The Crossing

The word Crossing here used is inspired by the notorious German musical term "*Durchführung*" (see GROVE, 1879, p. 472), to which it tries to propose an English translation. In this model, the Crossing denotes a procedure in which formulas in different regions succeed each other without a concern for the maintenance of a tonal anchor, or better put, a procedure in which the passage between regions happens from a point to another point in such a manner as to be meaningful, in principle, only the relationship between the new region and the last heard one. The crossing represents the basic model for tonal instability, and it is above all traditionally used in the construction of Development sections and Transitions.

In the present proposal for an analytical methodology, the crossing is notated by placing underneath the horizontal line which delimits the formula not one but two symbols for the region, separated by the sign for diatonic (homograph) or enharmonic equivalence (see BITTENCOURT, 2013, p. 42). The first symbol, to be placed to the left, indicates in two levels (in other words, one simple symbol and one inside parentheses immediately below) the relationship that the new region maintains with the last one heard, placing the symbol for that relationship above the symbol for the last heard region, this last one inside the parentheses. The second symbol, to be placed to the right, indicates in only one level (in other words, without a notation in parentheses) the relationship that the new region maintains with the main tonality of the work as a whole. The purpose of this method is twofold: 1) the first notation reveals that which is meaningful in the modulatory motion by crossing, namely, the point-to-point relationship; and 2) the second notation serves to allow the identification of the new region in just one level in a future formula, thus avoiding the inconvenience of presenting the next relationship symbol with a notation in three or more levels. This can be accomplished because with the proposal for a functional symbology here adopted it is always possible to describe with only one level any kind of relationship between two regions (see figure 5 and BITTENCOURT 2013, p. 41).

	C	D♭	D	E♭	E	F	F♯	G♭	G	A♭	A	B♭	B
M	$^{+}\overline{\mathrm{T}}$	$^{\circ}\overline{\mathrm{S}}\mathrm{a}$	$\overline{\mathrm{DD}}$	$^{\circ}\overline{\mathrm{T}}\mathrm{r}$	$^{+}\overline{\mathrm{T}}\mathrm{a}^{+}$	$^{+}\overline{\mathrm{S}}$	$\overline{\mathrm{DD}}\mathrm{a}^{+}$	$^{\circ}\overline{\mathrm{SS}}\mathrm{a}$	$\overline{\mathrm{D}}$	$^{\circ}\overline{\mathrm{T}}\mathrm{a}$	$^{+}\overline{\mathrm{T}}\mathrm{r}^{+}$	$^{+}\overline{\mathrm{SS}}$	$\overline{\mathrm{D}}\mathrm{a}^{+}$
m	$^{\circ}\overline{\mathrm{T}}$	$^{\circ}\overline{\mathrm{S}}\mathrm{a}^{\circ}$	$^{+}\overline{\mathrm{S}}\mathrm{r}$	$^{\circ}\overline{\mathrm{T}}\mathrm{r}^{\circ}$	$^{+}\overline{\mathrm{T}}\mathrm{a}$	$^{\circ}\overline{\mathrm{S}}$	$\overline{\mathrm{DD}}\mathrm{a}$	$^{\circ}\overline{\mathrm{SS}}\mathrm{a}^{\circ}$	$\overline{\mathrm{D}}^{\circ}$	$^{\circ}\overline{\mathrm{T}}\mathrm{a}^{\circ}$	$^{+}\overline{\mathrm{T}}\mathrm{r}$	$^{\circ}\overline{\mathrm{SS}}$	$\overline{\mathrm{D}}\mathrm{a}$

Figure 5. Table of relationships between the key of C major and all the other regions, measured in the most direct way possible (adapted from BITTENCOURT, 2013, p. 41).

Figure 6 illustrates the analysis process for a fragment of crossing. In this example, the analytical nomenclature tries to reveal and graph in symbols the sequenced repetition of the same point-to-point modulatory operation, always moving towards the region of the Tr° of the last one heard, thus causing a motion through an ascending cycle of minor thirds (or better put, via chromatic mediant relationships).

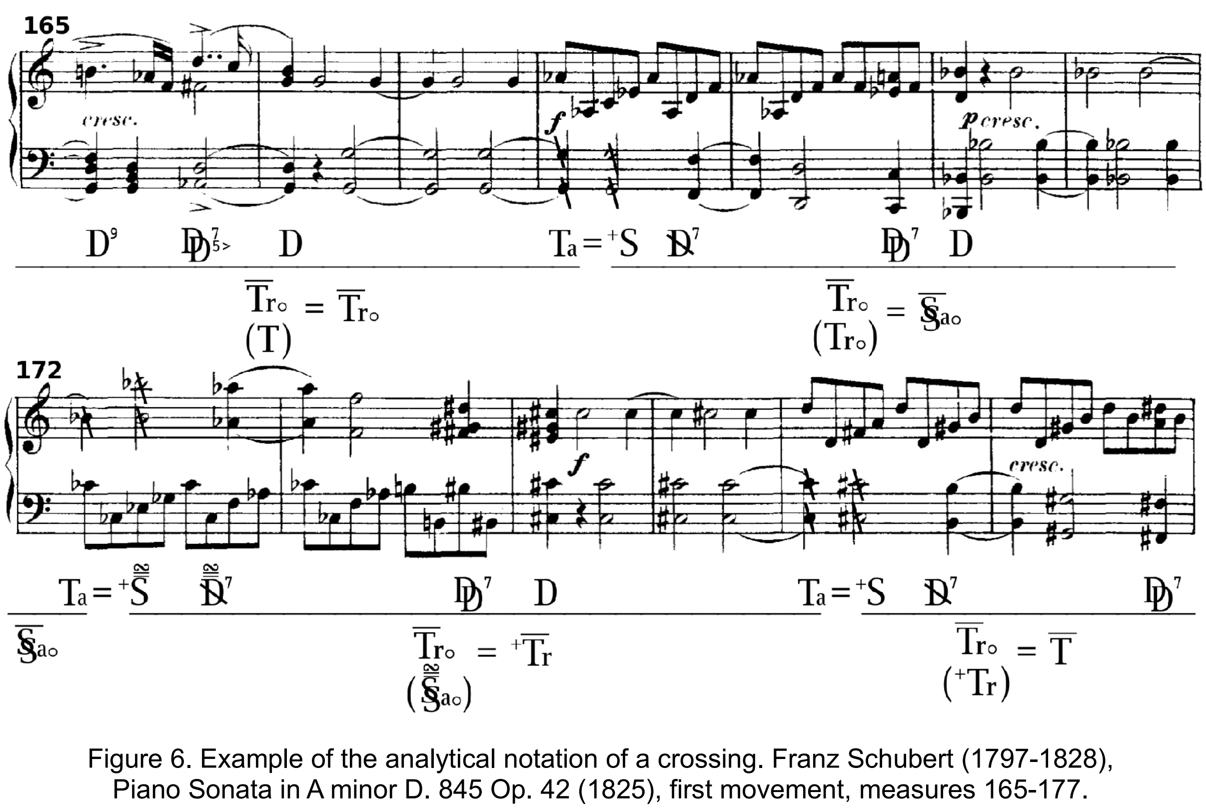

Figure 6. Example of the analytical notation of a crossing. Franz Schubert (1797-1828), Piano Sonata in A minor D. 845 Op. 42 (1825), first movement, measures 165-177.

3.1.5. The Wander

In this present proposal, the Wander is defined as a radical procedure in which the feeling for a tonal center is obliterated by means of a progression containing mainly dominant harmonic structures, each one pointing to a different region or even to several different regions, as in the case of the use of dominants with enharmonizable symmetrical structure (see "vagrant harmony" in SCHOENBERG, 1969, p. 44). In a certain way, the obliteration of center happens because of the quick changes and the excess and multiplicity of center indications. In short, this concept is basically analogous to the idea of "roving harmony" developed by Schoenberg (see SCHOENBERG, 1969, p. 3). Functionally, every dominant in the passage forms alone its own incomplete tonal formula, and in a certain way it is because of this that, in this case, the sequence of chords cannot "unmistakably express a region or a tonality" (SCHOENBERG, 1969, p. 3). Nonetheless, the end result of this is that as an ensemble the harmonic structures end up by forming a huge prolongation of the dominant area of a formula, in the manner of a sequence of extraordinary dominants, just waiting for one of the dominants to resolve into its expected target, thus halting the process of wander. This procedure is generally of short duration and it usually appears as a radical intensification of the modulatory movements of a crossing, specially inside Development sections. Its ending is commonly and naturally a half cadence which reconquers, if not a tonal anchor, at least a temporary center suitable for the start of new crossing procedures.

The wander generally involves the connection of several dominants via Chromatic Transformation, also known as Transformational Affinity (see BITTENCOURT, 2013, p. 42-43). Two harmonic structures are related by Transformational Affinity if a variant of the first structure (with added ninth, with chromatized fifth, with root omission, etc.) is identical to a variant of the second, either in a diatonic (homograph) manner, or in an enharmonic one (see BITTENCOURT, 2013, p. 42). This procedure is capable of easily transforming a dominant into another by means of a combination of chromatic and diatonic parsimonious voice-leading motions, what is certainly an evolution of the "law of the shortest way" preached by Bruckner (see SCHOENBERG, 1978, p. 39). This kind of relationship (indicated in the symbology proposal here adopted by the sign "≈") is specially found between the main dominant and the dominants of the relative and anti-relative substitutes of the tonic, either in a directly diatonic way or in a borrowed one. This fact leads us to believe that such chord progressions were "learned", so to speak, from the connections between dominants traditionally practiced in deceptive tonal formulas which involved a tonicization of the surprising element, as in the sequences [V7 – V7/vi – vi], [V7 – V7/iii – iii], for example, as well as their modal variants, either direct or borrowed. Such types of connection between dissonant chords, made possible by the creative maintenance of basic natural paradigms of voice leading, were already rather tentatively mapped by nineteenth-century theorists such as Weitzmann (see "deceptive progression" in RUDD, 1992, p. 65-66) and Reicha (see "exceptional resolutions of dissonant chords" in REICHA, 1890, p. 154, and REICHA, 1830, p. 2-5).

In the present proposal for an analytical methodology, the wander is graphed using the same principles utilized in the notation of crossings, but with the horizontal line indicating not exactly the frontiers of the tonal formulas but instead indicating tonal contextualization areas for the graphed harmonic structures. Thus, depending on what is most convenient for the analysis of the specific music fragment at hand, the harmonic structures will be contextualized: a) in the region last formed before the wandering process, or in the region finally conquered at the end of the wander, with the contradictory dominants marked as if they were tonicizations without targets or even marked as extraordinary dominants; or b) in several little formula stages, inasmuch as it is possible to decode and graph the typical paradigmatic origins of the progressions between dominants; or even c) in a combination of all the aforementioned principles. What should be kept in mind is that graphing exactly which are the centers pointed at by the dominants is not of much importance and is rarely pertinent. The important thing is to graph the logic and manner in which the dominants convert into each other, mapping this act of prolongation of a tonal instability tension until the reconquering of a new center. The example from figure 7 shows an example of wander in which it is possible to observe clearly those connections by chromatic transformational affinity, noting specially the connections of dominants according to the voice leading found in the typical tonicized deceptive cadences [V7 – V7/vi – vi] (measures 137-138 and 141-142) and [V7 – V7/iii – iii] (measures 143-144).

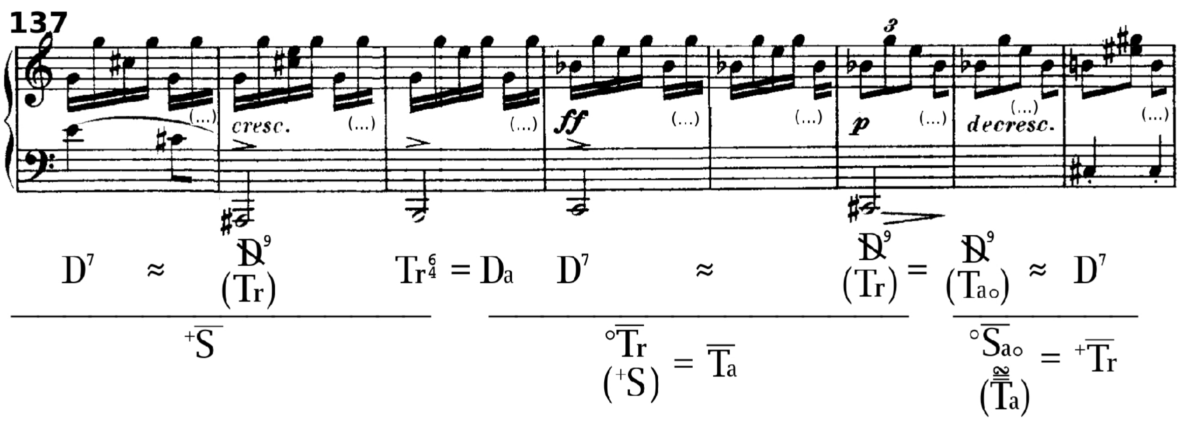

Figure 7. Example of the analytical notation of a wander.
Schubert, Piano Sonata in A minor D. 845 Op. 42, first movement, measures 137-144.

3.2. Complementary classification of modulations according the criterion of Facture

To complete this project of a taxonomy of modulations there is still the question of the identification of the Modulatory Facture. Here the criterion for classification is the manner in which the border area between two tonal formulas is managed, namely, the way in which the mixture and introduction of elements from different harmonic fields is demonstrated and constructed − hence the name "facture". This criterion of facture is always observable in all the previously seen types of modulation according to the modulatory impact criterion.

3.2.1. Modulation via Pivoted Facture

A first basic strategy to create a connection between regions is the use of a mediating element − traditionally known as a "pivot" − which is shared by the two harmonic fields involved. Strategically placed at the border area between the two tonal formulas, this mediating element is a harmonic structure which operates simultaneously and correctly − in other words, it participates *bona fide* of the unfurling of the tonal formulas in which it is included − both as the ending of the last heard formula and as the beginning of the next one (in the case of tonicization, we can consider the pair tonicization/tonicized-element as a "micro-formula"). Such strategy of interconnection between regions, performed by means of a harmonic element that undergoes, so to speak, a functional change of meaning, was already understood and described in detail at the beginning of the nineteenth century. For example, Gottfried Weber explains that "one and the same species of fundamental harmony may occur on more that one degree of a key, and indeed may belong at one time to one key, and at another time to another" (WEBER, 1851, p. 289), which works out the concept of "*Mehrdeutigkeit*" (multiple meaning) by Georg Joseph Vogler (1749-1814) (see DAMSCHRODER, 2008, p. 155-156). Created by Vogler and Weber (see BERNSTEIN, 2002, p. 778-788), the analytical technique of the traditional scale-degree harmony tries to demonstrate and illuminate exactly this question.

3.2.1.1. Pivoted Facture with a Direct Pivot

In this proposal, a pivot harmonic structure is considered to be of the "direct" type whenever it belongs to both the simple harmonic fields involved. By simple harmonic field, in this model, we mean the set of basic harmonic structures of a tonality (the pillars T, S, and D, also including, when in the minor mode, the modal variants +S e D° of melodic use mentioned in item 2.3.), plus their sixth (relative) substitutes and leading-tone (anti-relative) substitutes.

In this model, if the pivot exists named with identical spelling in both the simple harmonic fields involved, it is called a Direct Homograph Pivot, or more simply, a Diatonic Pivot. A modulation worked out using this type of pivot is the very same traditionally known as "diatonic modulation" (see ZAMACOIS, 1984, p. 202 or BRISOLLA, 2006, p. 77). The trick here is that one progresses in a diatonic manner through one region until it reaches the mediating element, from which it then proceeds also diatonically but now in the new region. In other words, this procedure prevents a direct clash between the conflicting elements of the diatonic spaces of the two regions involved. This double function of the pivot harmonic structure must be identified in the analyses by means of two functional markings for the pivot harmonic structure, separated by the sign for diatonic equivalence (=) (see BITTENCOURT, 2013, p. 42). The figure 8, between the measures 109 and 110, gives an example of this idea, analytically graphing the role of direct diatonic (or homograph) pivot that the triad of D minor performs in the border area between the formulas in D minor and E-flat major.

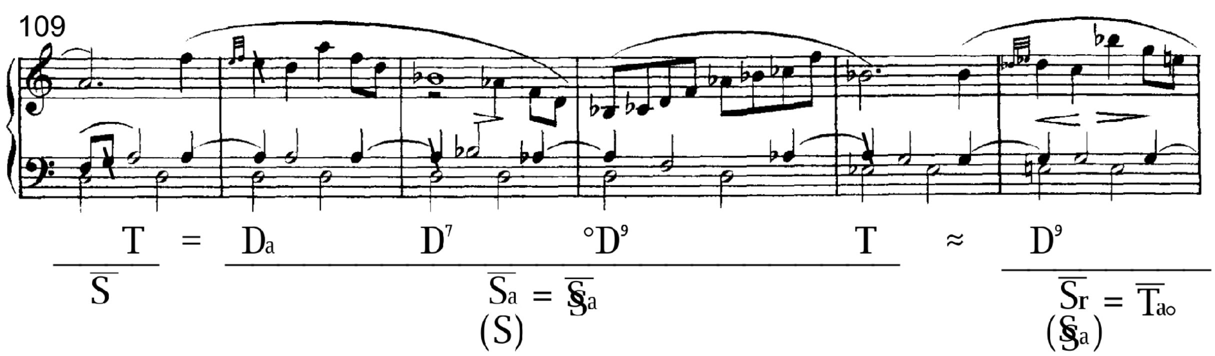

Figure 8. Example of the analytical notation of a pivoted modulation with a direct diatonic pivot, and of a non-pivoted modulation via a process of chromatic transformation.
Schubert, Piano Sonata in A minor D. 845 Op. 42, first movement, measures 109-114.

Figure 6 also includes examples of diatonic pivots on measures 168, 176 and also 172, disregarding the enharmony between the regions of G-flat minor and F-sharp minor. It is here important to remark that in cases of tonicization it is a bit superfluous and irrelevant to mark down pivots in the analytical notations because in such cases there is no real proposal for a change of region, something that would challenge or break the comprehension of the tonal formula. Such markings would only cloud the analytical fact to be graphed (namely, the very own tonicization).

It is also possible to accomplish the same aforementioned logic using as a pivot a harmonic structure that exists in both simple harmonic fields but named with different spellings. In this case, the pivot is called a Direct Enharmonic Pivot. A modulation performed with this type of pivot is basically the same one known traditionally as "enharmonic modulation" (see ZAMACOIS, 1990, p. 55 or BRISOLLA, 2006, p. 80). The double function of the direct enharmonic pivot must be identified in the analyses by means of the sign for enharmonic equivalence ($\cong$) (see BITTENCOURT, 2013, p. 42). Figure 9 shows an example of a direct enharmonic pivot (measure 23) which makes use of the notorious enharmonic equivalence between diminished seventh tetrads that lie a minor third apart, thus proposing a "shortcut" which interrupts and shortens a sequence of dominants in a cycle of fifths.

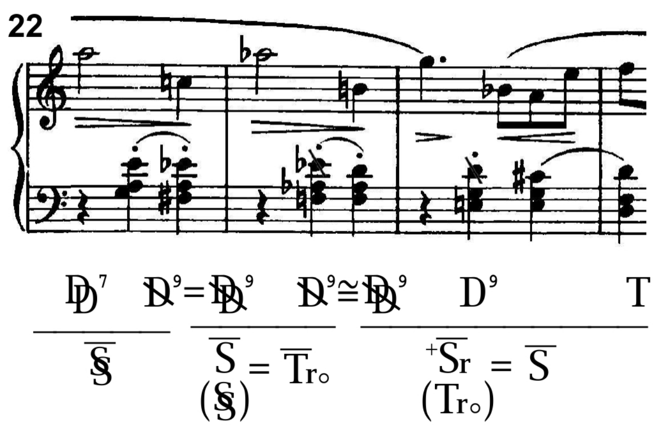

Figure 9. Example of the analytical notation of a direct enharmonic pivot.
Fréderic Chopin (1810-1849), Mazurka Op. 7 nº 2 in A minor, measures 22 to 25.

3.2.1.2. Pivoted Facture with a Borrowed Pivot

In this model, a pivot harmonic structure is of the "borrowed" kind whenever it does exist in both harmonic fields involved but only if we consider the extended versions of those fields, which include additions caused by processes of modal borrowing (a concept already sketched in the section "relationship to the minor subdominant", for example, in SCHOENBERG, 1978, p. 222).

In this process of expansion, to the simple harmonic field of a region will be added new harmonic structures which are obtained by means of combinations of operations of modal polarity inversion (major to minor or vice-versa), of sixth substitution and of leading-tone substitution, in several different orderings, performed on the basic pillar harmonic structures of the tonality (T, S, and D, including the modal variants of the minor mode mentioned in item 2.3.). Such combinations of operations, which where already partially mapped by Riemann in the nineteenth century (see RIEMANN, 1903, p. 88-106), have as end result the contextualization and inclusion in the same harmonic field of all 12 major triads and all 12 minor triads, in a hierarchical manner ranked in decreasing relationships of proximity, according to the number of operations involved: the bigger the number of operations needed to produce the new harmonic structure from one of the basic pillar triads of the tonality, the more hierarchically distant is the result.

According to this logic, the extended harmonic field of a region will be structured in four orbits of relationships in decreasing order of hierarchical proximity. The first orbit contains the very own basic pillars of tonality plus the resultant harmonic structures of the application on the pillars of just one operation of sixth or leading-tone substitution − that is, in this orbit revolves the very own simple harmonic field of the region. The second orbit contains the resultant structures of the application on the pillars of one operation of modal polarity inversion, optionally followed by one additional operation of sixth or leading-tone substitution − that is, in this orbit revolves the simple harmonic field of the modal parallel region. The third orbit contains the resultant structures of the application on the pillars of one operation of sixth or leading-tone substitution, followed by one operation of modal polarity inversion. The fourth orbit contains the resultant structures of the application on the pillars of a sequence of three operations, in this order: one operation of modal polarity inversion, one of sixth or leading-tone substitution and another one of modal polarity inversion − that is, the same principle which generated the third orbit is applied on the elements of the second orbit.

Regarding the process of construction of pivoted modulations, this expansion procedure implicates in a situation in which it is always possible to find pivot structures between any two tonalities, however odd and distant the proposed tonal relationships may sound. This ends up by providing the clues for the understanding of Weitzmann's statement that "between two consonant triads connected with natural voice leading there can always be shown an inner connection or relationship" (HARRISON, 1994, notes from pg. 1). It is easy to realize here that the consequence of such logic is the formation of the very own nineteenth-century concept of Extended Tonality.

In this model, similarly to the case of direct pivot types, if the borrowed pivot structure exists named with identical spellings in both harmonic fields involved only if we consider their extended versions, it is called a Borrowed Homograph Pivot. If the pivot exists named with different spellings in both harmonic fields involved only if we consider their extended versions, it is called a Borrowed Enharmonic Pivot. The double function of these pivots must also be identified in the analyses by means of the signs for diatonic equivalence (=), in the case of homography, and for enharmonic equivalence (≅), in the case of enharmony. Figure 10 gives an example of a borrowed homograph pivot of the second orbit, connecting two minor tonalities in chromatic mediant relationship (A minor and C minor) by means of an element from the harmonic field of C major, or rather, by means of a member of the second orbit of relationships of the extended harmonic field of C minor. Figure 2 (measure 8) also includes an example of a borrowed homograph pivot.

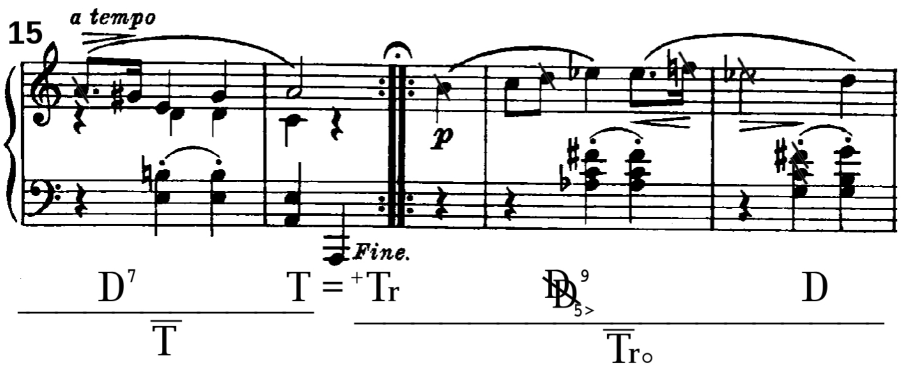

Figure 10. Example of the analytical notation of a borrowed homograph pivot of the second orbit.
Fréderic Chopin (1810-1849), Mazurka Op. 7 nº 2 in A minor, measures 15 to 18.

3.2.2. Modulation via Non-Pivoted Facture

A second strategy to perform the connection between regions is the use of a process of transformation that converts the last harmonic structure of the first formula into the first structure of the next formula by means of a chromatic transformation procedure (see item 3.1.5.), or rather, by means of a combination of chromatic and diatonic parsimonious voice-leading motions. While the strategy used in the pivoted modulation is, by the use of a common harmonic structure, to strive to avoid a direct clash between the conflicting elements of the different diatonic spaces involved, in the non-pivoted facture we value precisely the exposure of that clash by means of a process of "melodic distortion", which at the same time introduces a conflict − in the form of a diatonic contrast to the first region heard − and an unequivocal impulsion towards the concretization of a motion towards another harmonic field. This idea is basically analogous to the concept traditionally known as "chromatic modulation" (see ZAMACOIS, 1984, p. 208 or BRISOLLA, 2006, p. 80).

The occurrence of this method of junction between regions can be identified in the analyses via the sign for transformational affinity (≈), inserted between the two harmonic structures that form the border area of the two formulas involved (see BITTENCOURT, 2013, p. 42). Figure 8, between the measures 113 and 114, exemplifies this idea of modulation via non-pivoted facture, analytically graphing the conversion of the E-flat major triad (tonic in the first formula) into the C major triad (dominant in the second formula), by a process of chromatic transformation. It is again important to remark, and for the same aforementioned reasons, that the analytical notation of this relationship is usually irrelevant in cases of tonicization. Nonetheless, sometimes such markings may be pertinent, as in the case of wander found on figure 7.

4. Final Commentary

Currently in the process of formalization in a treatise of larger scope and breadth still under preparation, these analytical tools and the theoretical concepts here presented have demonstrated considerable pedagogical usefulness in classes of Harmony and Music Analysis, and they have always been practiced in a comparative fashion and counterposed to the classic tools of the Scale-Degree Theory and the traditional Functional Harmony. This pedagogical work has served as a testing ground for the development of this proposal of an analytical methodology, and it has been always showing − or even forcing − new paths to follow, exposing faults and nearsightednesses to be corrected, this as we verify the efficiency (or not) of the structural dissecting action of this proposed theoretical model on the works from the nineteenth-century repertoire. It is therefore natural that these propositions be subsequently modified and adjusted in a near future. In this paper, I tried to demonstrate the historical basis of this present proposal for a Taxonomy of Modulation, specially emphasizing its ability to graph the harmonic language of the nineteenth-century extended tonality.

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